DAY TWENTY NINE

Wave Optics

Learning & Revision for the Day

- Wavefront
- Interference of Light
- Young's Double Slit Experiment
- Coherent Sources
- Interference in Thin Films
- Diffraction
- Polarisation of Light
- Brewster's Law
- Law of Malus
- Polaroids

According to Huygens', light is a form of energy, which travels in the form of waves through a hypothetical medium 'ether'. The medium was supposed to be all pervading, transparent, extremely light, perfectly elastic and an ideal fluid.

Light waves transmit energy as well as momentum and travel in the free space with a constant speed of 3×10^8 ms^{-1}. However, in a material medium, their speed varies from medium to medium depending on the refractive index of the medium.

Wavefront

A wavefront is the locus of all those points (either particles) which are vibrating in the same phase. The shape of the wavefront depends on the nature and dimension of the source of light.

- In an isotropic medium, for a point source of light, the wavefront is spherical in nature.
- For a line (slit) source of light, the wavefront is cylindrical in shape.
- For a parallel beam of light, the wavefront is a plane wavefront.

Huygens' Principle

Every point on a given wavefront, acts as secondary source of light and emits secondary wavelets which travel in all directions with the speed of light in the medium. A surface touching all these secondary wavelets tangentially in the forward direction, gives the new wavefront at that instant of time.

Laws of reflection and refraction can be determined by using Huygens' principle.

Interference of Light

Interference of light is the phenomenon of redistribution of light energy in space when two light waves of same frequency (or same wavelength) emitted by two coherent sources, travelling in a given direction, superimpose on each other. If a_1 and a_2 be the amplitudes of two light waves of same frequency and ϕ be the phase difference between them, then the amplitude of resultant wave is given by

$$A_R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$$

and in terms of intensity of light,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi.$$

Condition for Constructive Interference

If at some point in space, the phase difference between two waves, $\phi = 0°$ or $2n\pi$ or path difference between two waves, $\Delta = 0$ or $n\lambda$, where n is an integer, then $A_R = a_1 + a_2$ or $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}$ is maximum. Such an interference is called **constructive interference**.

Condition for Destructive Interference

If at some point in space, the phase difference between two waves, $\phi = (2n-1)\pi$ or path difference, $\delta\Delta = (2n-1)\frac{\lambda}{2}$, then at such points $A_R = (a_1 - a_2)$ and $I_R = I_1 + I_2 - 2\sqrt{I_1 I_2}$ is minimum leading to a **destructive interference**.

Amplitude Ratio

$$\frac{I_{max}}{I_{min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2$$

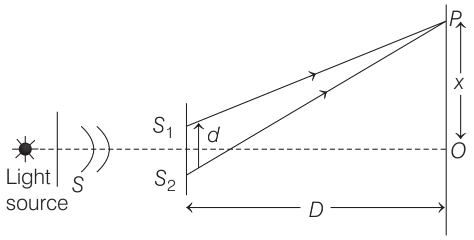
$$= \left[\frac{a_1 + a_2}{a_1 - a_2}\right]^2 = \left[\frac{r+1}{r-1}\right]^2$$

where, $r = \frac{a_1}{a_2}$ = amplitude ratio.

NOTE
- For identical sources, $I_1 = I_2 = I_0$
- For constructive interference, $I_{max} = 4I_0$ and $I = 4I_0 \cos^2 \frac{\phi}{2}$
- For destructive interference, $I_{min} = 0$

Young's Double Slit Experiment

The arrangement is shown in figure monochromatic light of one wavelength is used.



Young's experimental arrangement to produce interference pattern

Bright and dark fringes are formed on the screen with central point O behaving as the central bright fringe, because for O, the path difference $\Delta = 0$.

For light waves reaching a point P, situated at a distance x from central point Δ, the path difference,

$$\Delta = S_2P - S_1P = \frac{xd}{D}$$

Case I If $\frac{xd}{D} = n\lambda$, then we get nth bright fringe. Hence, position of bright fringes on the screen are given by the relation, $x = \frac{nD\lambda}{d}$.

Case II If $\frac{xd}{D} = (2n-1)\frac{\lambda}{2}$, then we get nth dark fringe.

Hence, for nth dark fringe,

$$x = \frac{(2n-1)D\lambda}{2d}$$

where, $n = 1, 2, 3, \ldots\ldots$.

Fringe Width

The separation between any two consecutive bright or dark fringes is called fringe width β.

Thus, $$\beta = \frac{D\lambda}{d}$$

and for a given arrangement, it is constant, i.e. all fringes are uniformly spaced.

Moreover, fringe width β is

(i) $\beta \propto D$, (ii) $\beta \propto \lambda$ and (iii) $\beta \propto \frac{1}{d}$.

Angular fringe width of interference pattern,

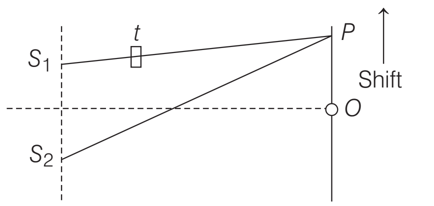
$$\alpha = \frac{\beta}{D} = \frac{\lambda}{d}$$

If in a given field of view n_1, fringes of light of wavelength λ_1 are visible and n_2 fringes of wavelength λ_2 are visible, then $n_1\lambda_1 = n_2\lambda_2$

NOTE
- If whole apparatus of Young's double slit experiment is immersed in a transparent medium of refractive index n_m, then fringe width in the medium, $\beta_m = \frac{D\lambda}{n_m d}$.

Displacement of Fringe Pattern

When a thin transparent plate is introduced in the path of one of the interfering waves trains, it is found that the entire fringe pattern is shifted through a constant distance. This shift takes place towards the wave train, in the path of which the plate is introduced.



Young's double slit experiment

The fringe width of the patterns remains same. Effective optical path that is equivalently covered in air is $S_1P + t(\mu - 1)$.

Thus, the path difference $= S_2P - S_1P - t(\mu - 1)$

$$= \frac{xd}{D} - t(\mu - 1)$$

[Optical path = refractive index × width of the material]

$\therefore$ Extra path $= \mu t - t = (\mu - 1)t$

Coherent Sources

Two light sources are said to be coherent, if they emit light of exactly same frequency (or wavelength), such that the originating phase difference between the waves emitted by them is either zero or remains constant. For sustained interference pattern, the interfering light sources must be coherent one.

There are two possible techniques for obtaining coherent light sources.

- In division of wavefront technique, we divide the wavefront emitted by a narrow source in two parts by reflection, refraction or diffraction.
- In division of amplitude technique, a single extended light beam of large amplitude is splitted into two or more waves by making use of partial reflection or refraction.

NOTE • Two independent sources of light can never be coherent. Two light sources can be coherent only, if these have been derived from a single parental light source.

Interference in Thin Films

In white light thin films, whose thickness is comparable to wavelength of light, show various colours due to interference of light waves reflected from the two surfaces of thin film.

For interference in reflected light condition of constructive interference (maximum intensity),

$$\Delta = 2n_m\, t \cos r = (2n-1)\frac{\lambda}{2}$$

Condition of destructive interference (minimum intensity),

$$\Delta = 2n_m\, t \cos r = (2n)\frac{\lambda}{2}$$

For interference in refracted light condition of constructive interference (maximum intensity),

$$\Delta = 2n_m t \cos r = (2n)\frac{\lambda}{2}$$

Condition of destructive interference (minimum intensity),

$$\Delta = 2n_m t \cos r = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \dots .$$

Shift in Interference Pattern

If a transparent thin sheet of thickness t and refractive index n_m is placed in the path of one of the superimposing waves (say in front of slit S_2 of Young's double slit experiment), then it causes an additional path difference due to which interference pattern shifts.

- Additional path difference due to sheet $= (n_m - 1)\,t$
- Fringe shift $= \frac{D}{d}(n_m - 1)t = \frac{\beta}{\lambda}(n_m - 1)t$
- If due to presence of thin film, the fringe pattern shifts by n fringes, then

$$n = \frac{(n_m - 1)\,t}{\lambda} \text{ or } t = \frac{n\lambda}{(n_m - 1)}$$

Shift is independent of the order of fringe and wavelength.

NOTE • **Fresnel's biprism** is a device to produce coherent sources by division of wavefront,

$$d = 2a(n-1)\alpha$$

The distance between the coherent sources and screen,

$$D = a + b$$

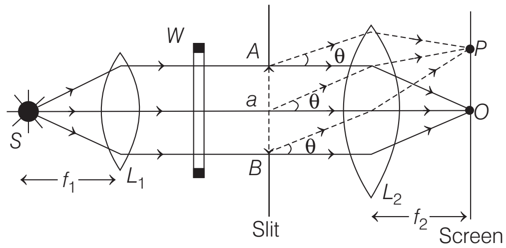
The fringe width is given by $\beta = \frac{D\lambda}{d} = \frac{\lambda(a+b)}{2a(n-1)\alpha}$

Diffraction

Diffraction of light is the phenomenon of bending of light around the edges of an aperture or obstacle and entry of light even in the region of geometrical shadow, when size of aperture or obstacle is comparable to wavelength of light used. Diffraction is characteristic of all types of waves. Greater the wavelength, more pronounced is the diffraction effect. It is due to this reason that diffraction effect is very commonly observed in sound.

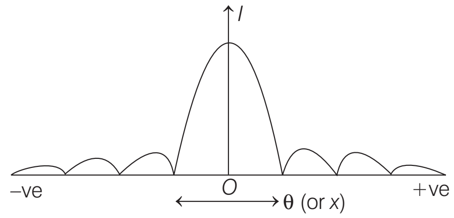
Diffraction due to a Single Slit

Fraunhofer's arrangement for studying diffraction at a single narrow slit (width $= a$) is shown in adjoining figure. Lenses L_1 and L_2 are used to render incident light beam parallel and to focus parallel light beam.



Fresnel diffraction through a slit

As a result of diffraction, we obtain a broad, bright maxima at symmetrical centre point O and on either side of it, we get secondary diffraction maxima of successively falling intensity and poor contrast, as shown in figure.



Intensity curve

- Condition of diffraction minima is given by

$$a \sin\theta = n\lambda$$

where, $n = 1, 2, 3, 4, \dots .$

But the condition of secondary diffraction maxima is

$$a \sin\theta = (2n+1)\frac{\lambda}{2}$$

where, $n = 1, 2, 3, 4, \dots .$

- Angular position of nth secondary minima is given by

$$\sin\theta = \theta = n\frac{\lambda}{a}$$

and linear distance, $x_n = D\theta = \frac{nD\lambda}{a} = \frac{nf_2\lambda}{a}$

where, f_2 is focal length of lens L_2 and $D = f_2$.

- Similarly, for nth maxima, we have

$$\sin\theta = \theta = \frac{(2n+1)\lambda}{2a} \text{ and } x_n = \frac{(2n+1)D\lambda}{2a} = \frac{(2n+1)f_2\lambda}{2a}$$

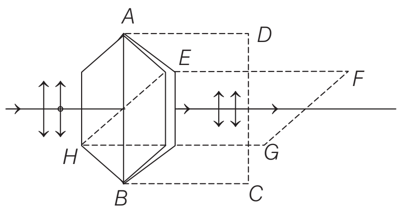
The angular width of central maxima is $2\theta = \frac{2\lambda}{a}$

or linear width of central maxima $= \frac{2D\lambda}{a} = \frac{2f_2\lambda}{2}$

NOTE
- The angular width of central maxima is double as compared to angular width of secondary diffraction maxima.

Polarisation of Light

- Light is an electromagnetic wave in which electric and magnetic field vectors very sinusoidally, perpendicular to each other as well as perpendicular to the direction of propagation of wave of light.
- The phenomenon of restricting the vibrations of light (electric vector) in a particular direction, perpendicular to the direction of wave motion is called **polarisation of light**. The tourmaline crystal acts as a **polariser**.



Polarisation of Light

Thus, electromagnetic waves are said to be polarised when their electric field vector are all in a single plane, called the plane of oscillation/vibration. Light waves from common sources are upolarised or randomly polarised.

Plane Polarised Light

The plane $ABCD$ in which the vibrations of polarised light are confined is called the **plane of vibration**. It is defined as The light, in which vibrations of the light (vibrations of electric vector) when restricted to a particular plane the light itself is called plane polarised light. The vibrations in a plane polarised light are perpendicular to the plane of polarisation.

NOTE
- Only transverse waves can be polarised. Thus, it proved that light waves are transverse waves.

Brewster's Law

According to this law, when unpolarised light is incident at an angle called polarising angle, i_p on an interface separating air from a medium of refractive index μ, then the reflected light is fully polarised (perpendicular to the plane of incidence), provided

$$\mu = \tan i_p$$

This relation represents Brewster's law. Note that the parallel components of incident light do not disappear, but refract into the medium, with the perpendicular components.

Law of Malus

When a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of cosine of angle (θ) between plane of transmission of analyser and polariser.

i.e. $$I \propto \cos^2\theta$$

If intensity of plane polarised light incidenting on analyser is I_0, then intensity of emerging light from analyser is $I_0\cos^2\theta$.

NOTE
- We can prove that when unpolarised light of intensity I_0 gets polarised on passing through a polaroid, its intensity becomes half, i.e. $I = \frac{1}{2}I_0$.

Polaroids

Polaroids are thin and large sheets of crystalline polarising material (made artifically) capable of producing plane polarised beams of large cross-section.

The important uses are

- These reduce excess glare and hence sun glasses are fitted with polaroid sheets.
- These are also used to reduce headlight glare of cars.
- They are used to improve colour contrast in old oil paintings.
- In wind shields of automobiles.
- In window panes.
- In three dimensional motion pictures.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The wavefront of a distant source of light is of shape

(a) spherical (b) cylindrical (c) elliptical (d) plane

2 Which of the following cannot be explained on the basis of wave nature of light?

(i) Polarisation (ii) Optical activity
(iii) Photoelectric effect (iv) Compton effect

(a) III and IV (b) II and III
(c) I and III (d) II and IV

3 A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is **→ AIEEE 2005**

(a) hyperbola (b) circle
(c) straight line (d) parabola

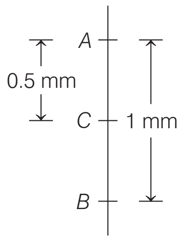
4 Two light rays having the same wavelength λ in vacuum are in phase initially. Then, the first ray travels a path L_1 through a medium of refractive index μ_1, while the second ray travels a path of length L_2 through a medium of refractive index μ_2. The two waves are then combined to observe interference.
The phase difference between the two waves is

(a) $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$ (b) $\frac{2\pi}{\lambda}(L_2 - L_1)$
(c) $\frac{2\pi}{\lambda}\left(\frac{L_1}{\mu_1} - \frac{L_2}{\mu_2}\right)$ (d) $\frac{2\pi}{\lambda}(\mu_2 L_1 - \mu_1 L_2)$

5 The Young's double slit experiment is performed with blue and with green light of wavelengths 4360 Å and 5460 Å respectively. If x is the distance of the 4th maxima from the central one, then

(a) $x_{(blue)} = x_{(green)}$
(b) $x_{(blue)} > x_{(green)}$
(c) $x_{(blue)} < x_{(green)}$
(d) $x_{(blue)} / x_{(green)} = 5460/4360$

6 In Young's double slit experiment, the length of band is 1mm. The fringe width is 0.021 mm. The number of fringes is



(a) 45 (b) 46
(c) 49 (d) 48

7 In a Young's double slit experiment, two coherent sources are placed 0.90 mm apart and the fringes are observed one metre away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of monochromatic light used will be

(a) 60×10^{-4} cm (b) 10×10^{-4} cm
(c) 10×10^{-5} cm (d) 6×10^{-5} cm

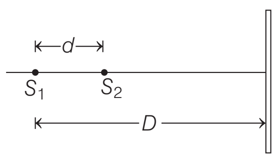
8 In Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case I_2, then the ratio $\frac{I_1}{I_2}$ is **→ AIEEE 2012**

(a) 4 (b) 2 (c) 1 (d) 0.5

9 A mixture of light consisting of wavelength 590 nm and an unknown wavelength, illuminates the Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is **→ AIEEE 2003**

(a) 885.0 nm (b) 442.5 nm
(c) 776.8 nm (d) 393.4 nm

10 Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be **→ JEE Main 2013**



(a) points (b) straight lines
(c) semi-circle (d) concentric circles

11 The source that illuminates the double-slit in 'double-slit interference experiment' emits two distinct monochromatic waves of wavelength 500 nm and 600 nm, each of them producing its own pattern on the screen. At the central point of the pattern when path difference is zero, maxima of both the patterns coincide and the resulting interference pattern is most distinct at the region of zero path difference.

But as one moves out of this central region, the two fringe systems are gradually out of step such that maximum due to one wave length coincides with the minimum due to the other and the combined fringe system becomes completely indistinct. This may happen when path difference in nm is

→ JEE Main (Online) 2013

(a) 2000 (b) 3000 (c) 1000 (d) 1500

12 In Young's double slit experiment, the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, I/I_0 is equal to **→ AIEEE 2007**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

13 In Young's double slit experiment, the intensity at a point is 1/4 of the maximum intensity. Angular position of this point is **→ AIEEE 2005**

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$ (c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

14 The first diffraction minimum due to single slit diffraction is θ, for a light of wavelength 5000 Å. If the width of slit is 1×10^{-4} cm. Then, the value of θ is

(a) 30° (b) 45° (c) 60° (d) 15°

15 A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of incident beam. At the first maximum of the diffraction pattern, the phase difference between the rays coming from the edges of the slit is

(a) 0 (b) $\pi/2$ (c) π (d) 2π

16 In Fraunhofer diffraction experiment, L is the distance between screen and the obstacle, b is the size of obstacle and λ is wavelength of incident light. The general condition for the applicability of Fraunhofer diffraction is

(a) $\frac{b^2}{L\lambda} >> 1$ (b) $\frac{b^2}{L\lambda} = 1$ (c) $\frac{b^2}{L\lambda} << 1$ (d) $\frac{b^2}{L\lambda} \neq 1$

17 In a Fraunhofer diffraction experiment at a single slit using a light of wavelength 400 nm, the first minimum is formed at an angle of 30°. The direction θ of the first secondary maximum is given by

(a) $\sin^{-1}\left(\frac{2}{3}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$

(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\tan^{-1}\left(\frac{2}{3}\right)$

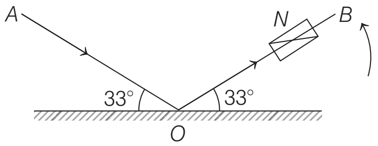
18 An unpolarised beam of light is incident on a group of four polarising sheets which are arranged in such a way that the characteristic direction of each polarising sheet makes an angle of 30° with that of the preceding sheet. The percentage of incident light transmitted by the first polariser will be

(a) 100% (b) 50% (c) 25% (d) 125%

19 A beam of ordinary unpolarised light passes through a tourmaline crystal C_1 and then it passes through another tourmaline crystal C_2 which is oriented such that its principal plane is parallel to that of C_2. The intensity of emergent light is I_0. Now, C_2 is rotated by 60° about the ray. The emergent ray will have an intensity

(a) $2I_0$ (b) $I_0/2$ (c) $I_0/4$ (d) $I_0/\sqrt{2}$

20 A beam of light AO is incident on a glass slab ($\mu = 1.54$) in a direction as shown in the figure. The reflected ray OB is passed through a nicol prism. On viewing through a nicol prism, we find on rotating the prism that



(a) the intensity is reduced down to zero and remains zero
(b) the intensity reduces down somewhat and rises again
(c) there is no change in intensity
(d) the intensity gradually reduces to zero and then again increases

21 When an unpolarised light of intensity I_0 is incident on a polarising sheet, the intensity of the light which does not get transmitted is **→ AIEEE 2005**

(a) $\frac{1}{2}I_0$ (b) $\frac{1}{4}I_0$ (c) zero (d) I_0

22 Two beams, A and B of plane polarised light with mutually perpendicular planes of polarisation are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then I_A/I_B equals

→ JEE Main 2014

(a) 3 (b) $\frac{3}{2}$ (c) 1 (d) $\frac{1}{3}$

23 A ray of light is incident on the surface of a glass plate of refractive index 1.732 at the polarising angle. The angle of refraction of the ray is

(a) 45° (b) 60° (c) 15° (d) 30°

24 A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented, so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is **→ JEE Main 2013**

(a) I_0 (b) $I_0/2$
(c) $I_0/4$ (d) $I_0/8$

Direction (Q. Nos. 25-28) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below*:

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

25 **Statement I** The thick film shows no interference pattern. Take thickness of the order of a few cms.

Statement II For interference pattern to be observed path difference between two waves is of the order of few wavelengths.

26 **Statement I** To observe diffraction of light, the size of obstacle/aperture should be of the order of 10^{-7} m.

Statement II 10^{-7}m is the order of wavelength of the visible light.

27 **Statement I** For a given medium, the polarising angle is 60°. The critical angle for this medium is 35°.

Statement II $\mu = \tan i_p$.

28 **Statement I** In Young's double slit experiment, the number of fringes observed in the field of view is small with longer wavelength of light and is large with shorter wavelength of light.

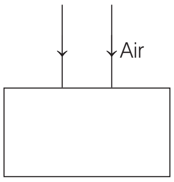
Statement II In the double slit experiment the fringe width depends directly on the wavelength of light.

→ **JEE Main (Online) 2013**

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A parallel beam of light of intensity I_0 is incident on a glass plate, 25% of light is reflected by upper surface and 50% of light in reflected from lower surface. The ratio of maximum to minimum intensity in interference region of reflected ray is

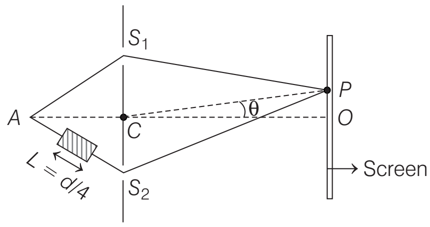


(a) $\left(\dfrac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (b) $\left(\dfrac{\frac{1}{4}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (c) $\dfrac{5}{8}$ (d) $\dfrac{8}{5}$

2 White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance d (>> b) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are

(a) $\lambda = \dfrac{2b^2}{3d}$ (b) $\lambda = \dfrac{b^2}{3d}$ (c) $\lambda = \dfrac{2b^2}{d}$ (d) $\lambda = \dfrac{3b^2}{d}$

3 A small transparent slab containing material of $\mu = 1.5$ is placed along AS_2 (figure). What will be the distance from O of the principal maxima and of the first minima on either side of the principal maxima obtained in the absence of the glass slab.



$AC = CO = D, S_1C = S_2C = d << D$

(a) $\dfrac{5}{\sqrt{235}}$ below point O (b) $\dfrac{5}{\sqrt{231}}$ below point O
(c) $\dfrac{5}{\sqrt{220}}$ below point O (d) $\dfrac{5}{\sqrt{110}}$ below point O

4 n identical waves each of intensity I_0 interfere with each other. The ratio of maximum intensities if the interference is (i) coherent and (ii) incoherent is

→ **JEE Main (Online) 2013**

(a) n^2 (b) $\dfrac{1}{n}$ (c) $\dfrac{1}{n^2}$ (d) n

5 In a Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by → **AIEEE 2012**

(a) $\dfrac{I_m}{9}(4+5\cos\phi)$ (b) $\dfrac{I_m}{3}\left(1+2\cos^2\dfrac{\phi}{2}\right)$
(c) $\dfrac{I_m}{5}\left(1+4\cos^2\dfrac{\phi}{2}\right)$ (d) $\dfrac{I_m}{9}\left(1+8\cos^2\dfrac{\phi}{2}\right)$

6 An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film? **→ AIEEE 2012**

(a) 7.2 m (b) 2.4 m (c) 3.2 m (d) 5.6 m

7 In a YDSE, bichromatic light of wavelength 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is **→ AIEEE 2004**

(a) 4 mm (b) 5.6 mm (c) 14 mm (d) 28 mm

8 The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double slit experiment, is **→ AIEEE 2004**

(a) infinite (b) five (c) three (d) zero

9 Unpolarised light of intensity I passes through an ideal polariser A. Another identical polariser B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now, another identical polariser C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polariser A and C is **→ JEE Main 2018**

(a) 0° (b) 30° (c) 45° (d) 60°

10 The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.) **→ JEE Main 2018**

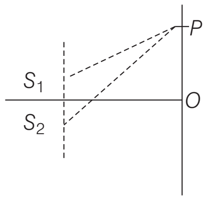
(a) 25 μm (b) 50 μm (c) 75 μm (d) 100 μm

11 The box of a pin hole camera of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when **→ JEE Main 2016 (Offline)**

(a) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$ (b) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(c) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$ (d) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$

12 In the YDSE apparatus shown in figure Δx is the path difference between S_1P and S_2P.



Now a glass slab is introduced in front of S_2, then match the following columns.

	Column I		Column II
A	Δx at P will	1.	increase
B	Fringe width will	2.	decrease
C	Fringe pattern will	3.	remain same
D	Number of fringes between O and P will	4.	shift upward
		5.	shift downward

	A	B	C	D
(a)	1	3	5	3
(b)	2	3	5	3
(c)	3	4	1	3
(d)	5	5	1	2

ANSWERS

SESSION 1

1 (d)	**2** (a)	**3** (a)	**4** (a)	**5** (c)	**6** (c)	**7** (d)	**8** (b)	**9** (b)	**10** (d)
11 (d)	**12** (d)	**13** (c)	**14** (a)	**15** (d)	**16** (c)	**17** (b)	**18** (b)	**19** (c)	**20** (d)
21 (a)	**22** (d)	**23** (d)	**24** (c)	**25** (a)	**26** (a)	**27** (a)	**28** (a)		

SESSION 2

1 (a)	**2** (b)	**3** (b)	**4** (d)	**5** (d)	**6** (d)	**7** (d)	**8** (b)	**9** (c)	**10** (a)
11 (c)	**12** (a)								

Hints and Explanations

SESSION 1

1 When the point source or linear source of light is at very large distance, wavefronts are plane wavefronts.

2 Photoelectric effect and Compton effect cannot be explained on the basis of wave nature of light while polarisation and optical activity can be explained.

3 Shape of interference fringes formed on a screen is hyperbolic in nature.

4 Optical path for 1st ray $= \mu_1 L_1$

Optical path for 2nd ray $= \mu_2 L_2$

$\therefore$ Path difference $= (\mu_1 L_1 - \mu_2 L_2)$

Now, phase difference

$$= \frac{2\pi}{\lambda} \times (\text{path difference})$$

$$= \frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$$

5 Distance of nth maxima,

$$x = n\lambda \frac{D}{d} \propto \lambda$$

As, $\lambda_b < \lambda_g$

$\therefore$ $x_{\text{blue}} < x_{\text{green}}$

6 The number of fringes on either side of C of screen is

$$n_1 = \left[\frac{AC}{\beta}\right] = \left[\frac{0.5}{0.021}\right] = [23.8] \approx 24$$

Total number of fringes

$= 2n_1 +$ fringe at centre $= 2n_1 + 1$

$= 2 \times 24 + 1 = 48 + 1 = 49$

7 $x = (2n - 1)\dfrac{\lambda}{2}\dfrac{D}{d}$

or $\lambda = \dfrac{2xd}{(2n-1)D}$

$$= \frac{2 \times 10^{-3} \times 0.9 \times 10^{-3}}{(2 \times 2 - 1) \times 1}$$

$= 6 \times 10^{-7}$ m $= 6 \times 10^{-5}$ cm

8 For coherent sources

$I_1 = 4I_0 \cos^2 \phi/2 = 4I_0$

For incoherent sources

$I_2 = I_0 + I_0 = 2I_0$

$\therefore$ $\dfrac{I_1}{I_2} = 2$

9 We have, $\dfrac{3D\lambda_k}{d} = \dfrac{4D\lambda_\mu}{d}$

where, λ_k is the known wavelength and λ_μ is the unknown wavelength. Thus, we get

$$\lambda_\mu = \frac{3\lambda_k}{4} = \frac{3}{4} \times 590 = 442.5 \text{ nm}$$

10 It will be concentric circles.

11 $\because n\lambda_1 = \left(n + \dfrac{1}{2}\right)\lambda_1$

$\Rightarrow$ $n \times 500 \times 10^{-9}$

$$= \left(n + \frac{1}{2}\right) \times 600 \times 10^{-9}$$

$\Rightarrow$ $n = \dfrac{3}{4}$

Now from the formula, $\Delta x = n\lambda$

or $\left(n + \dfrac{1}{2}\right)\lambda$

we get, $\Delta x = 1500$ nm

12 Phase difference

$$= \frac{2\pi}{\lambda} \times \text{path difference}$$

i.e. $\phi = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{6} = \dfrac{\pi}{3}$

As, $I = I_{\max} \cos^2\left(\dfrac{\phi}{2}\right)$

or $\dfrac{I}{I_{\max}} = \cos^2\left(\dfrac{\phi}{2}\right)$

or $\dfrac{I}{I_0} = \cos^2\left(\dfrac{\pi}{6}\right) = \dfrac{3}{4}$

13 $I = I_{\max} \cos^2\left(\dfrac{\phi}{2}\right)$

$\therefore$ $\dfrac{I_{\max}}{4} = I_{\max} \cos^2\left(\dfrac{\phi}{2}\right)$

$\cos\dfrac{\phi}{2} = \dfrac{1}{2}$ or $\dfrac{\phi}{2} = \dfrac{\pi}{3}$

$\therefore$ $\phi = \dfrac{2\pi}{3} = \left(\dfrac{2\pi}{\lambda}\right)\Delta x$ ...(i)

where, $\Delta x = d \sin\theta$

Substituting in Eq. (i), we get

$$\sin\theta = \frac{\lambda}{3d}$$

or $\theta = \sin^{-1}\left(\dfrac{\lambda}{3d}\right)$

14 The distance of first diffraction minimum from the central principal maximum is

$$x = \frac{D\lambda}{d}$$

$$\frac{x}{D} = \frac{\lambda}{d}$$

$\Rightarrow$ $d = \dfrac{\lambda}{\sin\theta}$

$\Rightarrow$ $\sin\theta = \dfrac{\lambda}{d} = \dfrac{5000 \times 10^{-8}}{1 \times 10^{-4}}$

$= 0.5 = \sin 30°$

$\Rightarrow$ $\theta = 30°$

15 The phase difference (ϕ) between the wavelets from the top edge and the bottom edge of the slit is $\phi = \dfrac{2\pi}{\lambda}(d \sin\theta)$

where, d is the slit width. The first minima of the diffraction pattern occurs at $\sin\theta = \dfrac{\lambda}{d}$,

So, $\phi = \dfrac{2\pi}{\lambda}\left(d \times \dfrac{\lambda}{d}\right) = 2\pi.$

16 The general condition for Frounhofer diffraction is $\dfrac{b^2}{L\lambda} << 1.$

17 For first diffraction minimum,

$$a \sin\theta = \lambda$$

$\Rightarrow$ $a = \dfrac{\lambda}{\sin\theta}$

For first secondary maximum,

$$a \sin\theta' = \frac{3\lambda}{2}$$

or $\sin\theta' = \dfrac{3\lambda}{2} \times \dfrac{1}{a} = \dfrac{3\lambda}{2} \times \dfrac{\sin\theta}{\lambda}$

$$= \frac{3}{2} \times \sin 30° = \frac{3}{4}$$

or $\theta' = \sin^{-1}\left(\dfrac{3}{4}\right)$

18 First polariser just polarises the unpolarised light. Therefore, intensity of polarised light transmitted from first polariser is

$$\frac{1}{2}I_0 = 50\% \, I_0$$

19 Intensity of light from $C_2 = I_0$

On rotating through 60°,

$$I = I_0 \cos^2 60°$$

$$= I_0\left(\frac{1}{2}\right)^2 = I_0/4$$

20 As, $i_p = \tan^{-1}(1.54) = 57°$

and in the figure given in question

$i = 90° - 33° = 57° = i_p$

$\therefore$ Reflected light along OB is plane polarised. On rotating the nicol prism, intensity gradually reduces to zero and then increases again.

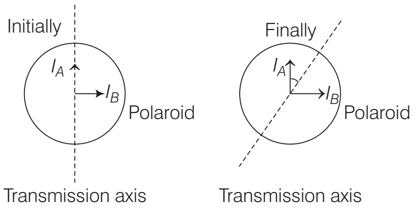
21 $I = I_0 \cos^2\theta$

Intensity of polarised light $= \dfrac{I_0}{2}$

$\therefore$ Intensity of untransmitted light

$$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

22 By law of Malus i.e. $I = I_0 \cos^2 \theta$



Now, $I_{A'} = I_A \cos^2 30°$

$I_{B'} = I_B \cos^2 60°$

As, $I'_A = I'_B$

$I_A \cos^2 30° = I_B \cos^2 60°$

$\Rightarrow \quad I_A \dfrac{3}{4} = I_B \dfrac{1}{4} \Rightarrow \dfrac{I_A}{I_B} = \dfrac{1}{3}$

23 From Brewster's law,

$\mu = \tan \theta_p$

$1.732 = \tan \theta_p$

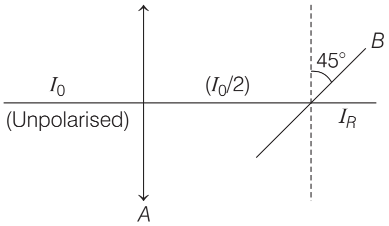
$\Rightarrow \quad \theta_p = \tan^{-1}(1.732) = 60°$

Since, the angle between i_p and r is 90° when the ray is incident at polarising angle, then

$\theta_p + r = 90°$

$r = 90° - \theta_p = 90° - 60° = 30°$

24 Relation between intensities is



$I_R = \left(\dfrac{I_0}{2}\right)\cos^2(45°) = \dfrac{I_0}{2} \times \dfrac{1}{2} = \dfrac{I_0}{4}$

25 For interference to occur, the path difference between two waves is of the order of few wavelength.

26 For diffraction to occur, the size of an obstacle/aperture is comparable to the wavelength of light wave. The order of wavelength of light wave is 10^{-7} m, so diffraction occurs.

27 From the relation,

$\mu = \tan i_p = \tan 60° = \sqrt{3}$ ...(i)

and $\sin C = \dfrac{1}{\mu} = \dfrac{1}{\sqrt{3}}$ [from Eq. (i)]

$\therefore \; C = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = \sin^{-1}(0.577) = 35°$

28 The number of fringe is smaller in case of larger wavelength is used while in case of smaller wavelength is used the number of fringe is larger.
Also, fringe width is given by

$\beta = \dfrac{\lambda D}{d} \Rightarrow \beta \propto \lambda$

SESSION 2

1 The intensity of light reflected from upper surface is

$I_1 = I_0 \times 25\% = I_0 \times \dfrac{25}{100} = \dfrac{I_0}{4}$

Intensity of transmitted light from upper surface is

$I = I_0 - \dfrac{I_0}{4} = \dfrac{3I_0}{4}$

$\therefore$ Intensity of reflected light from lower surface is

$I_2 = \dfrac{3I_0}{4} \times \dfrac{50}{100} = \dfrac{3I_0}{8}$

$\dfrac{I_{max}}{I_{min}} = \dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

$\dfrac{I_{max}}{I_{min}} = \dfrac{\left(\sqrt{\dfrac{I_0}{4}} + \sqrt{\dfrac{3I_0}{8}}\right)^2}{\left(\sqrt{\dfrac{I_0}{4}} - \sqrt{\dfrac{3I_0}{8}}\right)^2}$

$= \dfrac{\left(\dfrac{1}{2} + \sqrt{\dfrac{3}{8}}\right)^2}{\left(\dfrac{1}{2} - \sqrt{\dfrac{3}{8}}\right)^2}$

2 Path difference $= S_2P - S_1P$.
From the figure,

$(S_2P)^2 - (S_1P)^2 = b^2$

or $(S_2P - S_1P)(S_2P + S_1P) = b^2$

or $(S_2P - S_1P) = \dfrac{b^2}{2d}$

For dark fringes,

$\dfrac{b^2}{2d} = (2n+1)\dfrac{\lambda}{2}$

For $n = 0$, $\dfrac{b^2}{2d} = \dfrac{\lambda}{2}$ or $\lambda = \dfrac{b^2}{d}$

For $n = 1$, $\dfrac{b^2}{2d} = \dfrac{3\lambda}{2}$ or $\lambda = \dfrac{b^2}{3d}$

3 In case of transparent glass slab of refractive index μ, the path difference $= 2d \sin\theta + (\mu - 1)L$.
For the principal maxima, (path difference is zero)
i.e. $2d\sin\theta_0 + (\mu - 1)L = 0$

or $\sin\theta_0 = -\dfrac{L(\mu - 1)}{2d} = \dfrac{-L(0.5)}{2d}$

$[\therefore \; L = d/4]$

or $\sin\theta_0 = \dfrac{-1}{16}$

or $OP = D\tan\theta_0 = D\sin\theta_0 = \dfrac{-D}{16}$

For the first minima, the path difference is $\pm \dfrac{\lambda}{2}$

$\therefore \quad 2d\sin\theta_1 + 0.5L = \pm\dfrac{\lambda}{2}$

or $\sin\theta_1 = \dfrac{\pm\lambda/2 - 0.5L}{2d}$

$= \dfrac{\pm\lambda/2 - d/8}{2d}$

$= \dfrac{\pm\lambda/2 - \lambda/8}{2\lambda} = \pm\dfrac{1}{4} - \dfrac{1}{16}$

[$\because$ The diffraction occurs if the wavelength of waves is nearly equal to the slit width (d)]

On the positive side

$\sin\theta'_1 = +\dfrac{1}{4} - \dfrac{1}{16} = \dfrac{3}{16}$

On the negative side

$\sin\theta''_1 = -\dfrac{1}{4} - \dfrac{1}{16} = -\dfrac{5}{16}$

The first principal maxima on the positive side is at distance

$D\tan\theta'_1 = D\dfrac{\sin\theta'_1}{\sqrt{1 - \sin^2\theta'_1}}$

$= D\dfrac{3}{\sqrt{16^2 - 3^2}} = \dfrac{3D}{\sqrt{247}}$ above point O.

The first principal minima on the negative side is at distance

$D\tan\theta''_1 = \dfrac{5}{\sqrt{16^2 - 5^2}} = \dfrac{5}{\sqrt{231}}$ below point O.

4 **When interference is coherent**
When two waves of intensities I_1 and I_2 having a phase difference ϕ interfere, the resultant intensity is given as

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$ ...(i)

The intensity will maximum, then $\phi = 0$ or $\cos\phi = 1$
maximum intensity.

$\therefore \; I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$= (\sqrt{I_1} + \sqrt{I_1})^2$

In case, n identical waves each of intensities I_0 interfere,

$I_{max} = (\sqrt{I_0} + \sqrt{I_0} + \sqrt{I_0} + \ldots + n \text{ times})^2$

$= (n\sqrt{I_0})^2$ ...(i)

$\therefore \; I_{max} = n^2 I_0$ ...(ii)

When interference is incoherent
Since, the average value of $\cos\phi$, over a complete cycle is zero
The Eq. (i), becomes,

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \times 0$

$= I_1 + I_2$ ...(iii)

In case, n identical waves, each of intensities I_0 interfere,
Minimum intensity,

$I_{min} = I_0 + I_0 + I_0 + \ldots n$ times

$I_{min} = nI_0$ ...(iv)

$\therefore$ Ratio $\dfrac{I_{max}}{I_{min}} = \dfrac{n^2 I_0}{nI_0} = n$

5 Given, $a_1 = 2a_2$

$\Rightarrow \quad I_1 = 4I_2 = 4I_0$

$\therefore \quad I_m = (\sqrt{I_1} + \sqrt{I_2})^2 = (3\sqrt{I_2})^2$

$= 9I_2 = 9I_0 = I_0 = \frac{I_m}{9}$

Now, resultant intensity,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$= 4I_0 + I_0 + 2\sqrt{4I_0 I_0}\cos\phi$$

$$= 5I_0 + 4I_0\cos\phi$$

$$= \frac{I_m}{9}(5 + 4\cos\phi)$$

$$= \frac{I_m}{9}[1 + 4(1 + \cos\phi)]$$

$$= \frac{I_m}{9}(1 + 8\cos^2\phi/2)$$

$$\left[(1 + \cos\theta) = 2\cos^2\frac{\phi}{2}\right]$$

6 Shift in image position due to glass plate,

$$S = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{1}{1.5}\right) \times 1\text{ cm} = \frac{1}{3}\text{ cm}$$

For focal length of the lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} - \frac{1}{-240} = \frac{20 + 1}{240}$$

$\Rightarrow \quad f = \frac{240}{21}$ cm

Now, to get back image on the film, lens has to form image at $\left(12 - \frac{1}{3}\right)$ cm $= \frac{35}{3}$ cm such that the glass plate will shift the image on the film.

As, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f}$

$$= \frac{3}{35} - \frac{21}{240}$$

$$\frac{1}{u} = \frac{48 \times 3 - 7 \times 21}{1680} = -\frac{1}{560}$$

$\Rightarrow \quad u = -5.6$ m

7 Let nth minima of 400 nm coincides with mth minima of 560 nm, then

$$(2n - 1)\left(\frac{400}{2}\right) = (2m - 1)\left(\frac{560}{2}\right)$$

or $\quad \frac{2n - 1}{2m - 1} = \frac{7}{5} = \frac{14}{10} = \ldots$

i.e. 4th minima of 400 nm coincides with 3rd minima of 560 nm.

Location of this minima is,

$$Y_1 = \frac{(2 \times 4 - 1)(1000)(400 \times 10^{-6})}{2 \times 0.1}$$

$= 14$ mm

Next 11th minima of 400 nm will coincide with 8th minima of 560 nm.

Location of this minima is,

$$Y_2 = \frac{(2 \times 11 - 1)(1000)(400 \times 10^{-6})}{2 \times 0.1}$$

$= 42$ mm

$\therefore$ Required distance $= Y_2 - Y_1 = 28$ mm

8 For possible interference maxima of the screen, the condition is

$d\sin\theta = n\lambda \quad$ ...(i)

Given, $\quad d$ = slit width $= 2\lambda$

$\therefore \quad 2\lambda\sin\theta = n\lambda$

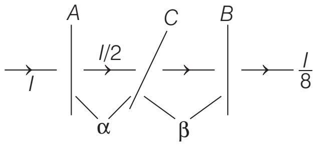
or $\quad 2\sin\theta = n$

The maximum value of $\sin\theta$ is 1, hence,

$n = 2 \times 1 = 2$

Thus, Eq. (i) must be satisfied by 5 integer values, i.e. $-2, -1, 0, 1, 2$. Hence, the maximum number of possible interference maxima is 5.

9



Using Malus's law, intensity available after $C = \frac{I}{2} \times \cos^2\alpha$

and intensity available after

$$B = \frac{I}{2}\cos^2\alpha \times \cos^2\beta$$

$$= \frac{I}{8} \text{ (given)}$$

So, $\frac{I}{2} \times \cos^2\alpha \cdot \cos^2\beta = \frac{I}{8}$

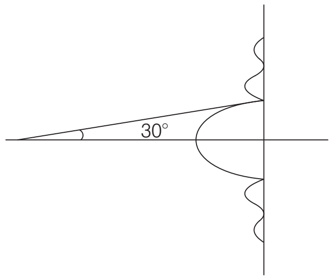
$\Rightarrow \quad \cos^2\alpha \cdot \cos^2\beta = \frac{1}{4}$

This is satisfied with $\alpha = 45°$

and $\beta = 45°$

So, angle between A and C is $45°$.

10 Angular width of diffraction pattern $= 60°$



For first minima,

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2},$$

[here, $a = 10^{-6}$ m, $\theta = 30°$]

$\Rightarrow \quad \lambda = 10^{-6} \times \sin 30°$

$\Rightarrow \quad \lambda = \frac{10^{-6}}{2}$ m

Now, in case of interference caused by bringing second slit,

$\therefore$ Fringe width,

$$\beta = \frac{\lambda D}{d}$$

[here, $\lambda = \frac{10^{-6}}{2}$ m, $\beta = 1$ cm $= \frac{1}{100}$ m,

d = ? and $D = 50$ cm $= \frac{50}{100}$ m]

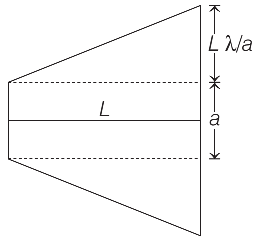
So, $\quad d = \frac{\lambda D}{\beta} = \frac{10^{-6} \times 50}{2 \times \frac{1}{100} \times 100}$

$= 25 \times 10^{-6}$ m

or $\quad d = 25\,\mu$m

11 In diffraction, first minima, we have

$$\sin\theta = \frac{\lambda}{a}$$



So, size of a spot,

$b = 2a + \frac{2L\lambda}{a} \quad$ ...(i)

Then, minimum size of a spot, we get

$$\frac{\partial b}{\partial a} = 0 \Rightarrow 1 - \frac{L\lambda}{a^2} = 0$$

$\Rightarrow \quad a = \sqrt{\lambda L} \quad$ ...(ii)

So, $b_{min} = 2\sqrt{\lambda L} + 2\sqrt{\lambda L}$

[by substituting the value of a from Eq. (ii) in Eq. (i)]

$= 4\sqrt{\lambda L}$

So, the radius of the spot,

$$b_{min} = \frac{4}{2}\sqrt{\lambda L} = \sqrt{4\lambda L}$$

12 From YDSE,

(A) Path difference $\Delta x = \frac{xd}{D}$

So, as x increases, Δx also increases.

(B) Fringe width $(\beta) = \frac{Dx}{d}$

independent of Δx.

(C) Fringe pattern will shift downward.

(D) β is constant, so number of fringes unaffected.